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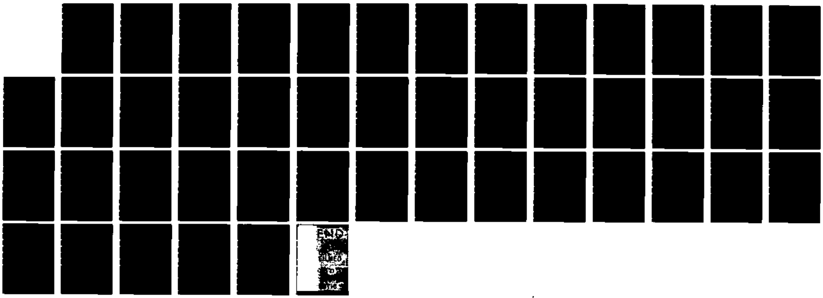

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FURTHER DEVELOPMENT OF IMPROVED METHODS

FOR

LARGER SCALE STURCTURAL SYNTHESIS

AFOSR-80-0194

by

Michael Pappas

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This report presents a general procedure for numerical design optimization base on the feasible direction finding problem (DFP) of Zoutendijk. The DFP is reformulated herein to allow treatment of multiple objective functions and efficient treatment of equality and inequality constraints. This new DFP formulation is combined with an adaptive move strategy to produce a Mathematical Progrmming (MP) algorithm applicable to a broad class of nonlinear MP problems with continuous variables including generalized mini-max problems occurring in control and structural system design. The application of this procedure to

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ABSTRACT

This report presents a general procedure for numerical design optimization based on the feasible direction finding problem (DFP) of Zoutendijk. The DFP is reformulated herein to allow treatment of multiple objective functions and efficient treatment of equality and inequality constraints. This new DFP formulation is combined with an adaptive move strategy to produce a Mathematical Programming (MP) algorithm applicable to a broad class of nonlinear MP problems with continuous variables including generalized mini-max problems occurring in control and structural system design. The application of this procedure to structural optimization is described in detail.

LIST OF FIGURES

TABLE OF CONTENTS

I. INTRODUCTION

The numerical design optimization problem is usually posed in Mathematical Programming (MP) form as:

Find $\bar{x}$ such that

$$f(x) = \min f(x) \tag{1}$$

$$g_j(\bar{x}) \leq 0 \qquad j = 1,2,\ldots J \tag{2}$$

$$h_k(\bar{x}) = 0 \qquad k = 1,2,\ldots K \tag{3}$$

where x is the continuous system design or control variable vector with components, x_i, $i = 1,2,\ldots I$ $f(x)$ the objective function and $g_j(x)$ and $h_k(x)$ the constraint functions. Numerous approaches to the solution of this problem are now is use [1].[1]

For many engineering design applications a single objective function of the form of Eq. (1) is inadequate to define the design objectives [2-4]. Reference [2] describes a pair of two variable bearing design problems with competing dual objectives which are treated by trade-off methods. Goal programming [5] is used in Ref. [3] to treat a metal removal optimization problem. Both [2] and [3] treat the multiple objective function problem by converting it to one of several single objective function forms to reduce the problem to the conventional MP form of Eqs. (1-3).

In design applications one often encounters a form of mini-max problem in that the determination of the objective or constraint functions given a set of design or control variables involves the solution of an optimization problem with respect to another to another variable set (state variables).

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[1]Numbers in brackets designate references cited at the end of this report.

This situation arises frequently in optimal control and structural optimization problems. Thus an MP formulation more general then that given by Eqs. (1-3) and a procedure for its solution, is needed for many applications.

Consider the important field of structural optimization. Extensive literature exists on numerical structural optimization [6]. It is well known that optimal structures are characterized by the frequent presence of multiple active (critical) behavior modes. Simultaneous failure mode concepts were widely used in optimal design [7] prior to the use of the more current Mathematical Programming (MP) and Optimality Criteria (OC) numerical optimization methods pioneered in structural applications by Schmit [8] and Venkayya [9] respectively.

Although multiple failure or behavioral modes are commonly considered in earlier works the treatment of mode coalesence is often inadquate. For example structural design MP formulations typically consider only a single general instability constraint [10-11] where several should be included. It may be seen from the contour maps of the buckling load surface given in [12] that several general buckling modes can be active in near optimal designs. When using a numerical technique to treat such problems one must consider all active behavior modes in the determination of search moves. If this is not done then moving so as to reduce or avoid a violation in only the most critical mode considered may result in an increase in a nearly critical mode that was not considered. This mode may then become critical after the move producing move failure.

When the behavior modes occur in the problem constraints multiple active modes may be treated conventionally by associating a separate constraint with each active mode since constrained numerical optimization techniques normally treat multiple constraints [13]. When, however, the objective function involves behavior, multiple active behavior modes produce a multiple objective mode mini-max problem. This situation was not recognized until recently [4]. Thus studies such as that of Ref. [14] involving optimal frequency separation considering only separation between only two modes will not in general yield an optimal solution since simultaneous separation of several mode pairs is required [4].

Most structural optimization problems requiring numerical methods are some variant of the mini-max problem. For example, weight optimization is usually subject to constraints on maximum stress or minimum buckling load. Frequently of course, the determination of the behavioral optima is trival and need not be explicity included in the general optimization problem. Often, however, it is not. Unfortunately, the literature, with few exceptions [12], contains little on the application of numerical procedures to the determination of behavioral optima.

Most general MP procedures require several hundred or even several thousand sets of constraint and objective function evaluations [15] to achieve an optimum. Such computational effort is prohibitively expensive in many applications which require considerable effort for each objective or constraint function evaluation. Thus, for example, one sees the development of special algorithms for the optimal design of structures modeled by Finite Elements utilizing the special properties of such problems in order to produce usable optimal design capabilities [6]. An effective general MP procedure for optimal design must be substantially more efficient than most existing MP procedures if current computationally demanding analysis techniques are to be exploited for optimal design. It usually makes little sense to optimize a design using inferior anlytical models because the more computationally demanding and more accurate methods are incompatable with optimal design procedures.

This report proposes and examines a general procedure for dealing with such problems. It describes a highly efficient, rigorous general procedure for the solution of an expanded MP problem applicable to a wide range of design optimization applications including those with the multiple objective functions and mini-max problem types. The method maintains the multiple objective function in its formulation and procedure and does not resort to subjective or other conversion procedures to reduce the multiple objective MP problem to one involving only a single objective function. Also described are various mini-max forms encountered in such problems and methods for treating these problem forms.

II. PROBLEM FORMULATION

The general design optimization problem may be written as: Find the optimal values $\bar{x}$ of the design variables x and the associated state variable vector t with components $\bar{t}_p$ p = 1,2...P so as to simultaneously minimize with respect to x all $f_q(x)$ where

$$f_q(x) = \max_t F_q(x,t) \qquad q = 1,2,\ldots Q \ . \tag{4}$$

This may be restated as: Find $\bar{x}$ and $\bar{t}$ such that

$$f_q(\bar{x},\bar{t}) = \operatorname{smin}_x \max_t Fq(x,t) . \tag{4a}$$

The minimizations of Eqs. (4) are subject to the conditions of Eqs. (2 & 3) where some or all of the constraints are of the form

$$g_j(x) = \max_t G_j(x,t) \leq 0 \tag{5}$$

$$h_k(x) = \max_t H_k(x,t) = 0 \tag{6}$$

It is convenient to remove the regional constraints from Eq. (3) by separating constraints of the form

$$x_i^{\ell} \leq x_i \leq x_i^u \tag{7}$$

$$t_p^{\ell} \leq t_p \leq t_p^u \tag{8}$$

where x_i^{ℓ} and x_i^u are the lower and upper limits respectively on design variable components x_i and t_p^{ℓ} and t_p^u the lower and upper limits on the state variable components t_p. This problem will be referred to here as the generalized mini-max problem. Equations (1-3) are a special case of Eqs. (4-6) where Q = 1 and no maximization with respect to the state variables is needed.

A variation of the mini-max objective function optimization problem of Eq. (4) or (4a) often stems from a maximum performance problem where the objective minimization is given by

Find $\bar{x}$ and $\bar{t}$ such that

$$f(\bar{x},\bar{t}) = \min_{x,q} \max_{t} F_q(x,t) \cdot \qquad (9)$$

This can be restated in the multiple objective function form of Eq. (4) for solution as will be shown in Section IV. Bronowicki et al [14] pose a frequency separation problem where a single objectiv function is of the form of Eq. (9) with q = 1,2 being identified with shell panel and general shell vibration respectively and the state variables with the vibration axial and circumferential wave mode integers. The coalesence of vibration modes as the optimum is approached requires simultaneous separation of several frequencies [4]. Thus one has the need for the consideration of several objective modes and an appropriate method for treatment of such problems. In the composite cylinder study of Ref. [16] over twenty buckling modes are active in some optimal designs.

The solution to the general problem of Eqs. (4-8) may be approached iterating the control or design variables

$$x^{r+1} = R(x^r) \qquad (10)$$

where R is the recursion relation defined by the search strategy. The maximization sub-problems of Eqs. (4-6) are solved at each point x^r.

III. GENERAL MINIMIZATION (CONTROL VARIABLE) PROCEDURE

In light of the fact that a general procedure for design optimization requires the treatment of constrained problems with multiple objective modes in addition to the conventional MP problem the idea presented in Ref. [4] is adapted to the procedure proposed here. The procedure of [4] utilizes a direct "Pattern" search coupled with the Zoutendijk direction finding problem (DFP) [17], modified to treat multiple objective modes. The DFP is further modified using the ideas explored in [18] so as to greatly improve its convergence power. The direction finding problem is further modified here to allow movement in the infeasible region. The symmetric penalty method of Refs. [11,19] is utilized here for the purpose of comparing feasible and infeasible points in the move strategy.

The procedure is given by the following steps:

1. Select an arbitrary initial point x_B^o and define $x_T^o = x_B^o$. With $r = 0$
2. evaluate $C_q(x_B^r)$ if not previously determined where

$$C_q(x) = f_q(x) + P_q(x) \tag{11}$$

$$P_q(x) = \max_{j,k} \{\lambda_{gj} \quad \langle g_j(x)\rangle \quad \text{or } \lambda_{qk} h_k(x)\} \tag{12}$$

$$\lambda_{gj} = \begin{cases} 2|\nabla f_q|/\nabla g_j \cdot \nabla f_q & g_j \leq \varepsilon_{j1} \\ K & g_j > \varepsilon_{j1} \end{cases} \tag{13}$$

and λ_{qk} similarly defined where $|A|$ is the magnitude of vector A, $\nabla\phi$ the gradient of scalar function ϕ, K_1 an arbitrary large positive number and ε_{j1} a band width parameter defining excess constraint violation [11,19]. The bracket function is defined as

$$\begin{aligned} <\phi> &= 0 \quad \phi < 0 \\ <\phi> &= 0 \quad \phi > 0 \end{aligned} \tag{14}$$

3. At point x_T^r set up the DFP and find σ and S^r so as to:

$$\text{maximize} \quad \sigma \tag{15}$$

subject to the conditions

$$S^r \cdot \nabla f_q^r + \sigma \leq 0 \qquad q = 1,2,\ldots Q \tag{16}$$

$$S^r \cdot \nabla g_j^r + g_j^r \leq 0 \qquad j \varepsilon J_A,\ g_j(x^r) > -\varepsilon_{j2}^r \tag{17}$$

$$S^r \cdot \nabla h_k + h_k^r = 0 \qquad k = 1,2,\ldots K \tag{18}$$

$$(S_i^\ell)^r \leq S_i^r \leq (S_i^u)^r \qquad i = 1,2,\ldots I \tag{19}$$

where ε_{j2}^r is a constraint activity band width parameter and $(S_i^\ell)^r$ and $(S_i^u)^r$ are upper and lower limits on S_i^r which are given by

$$S_i^\ell = \begin{cases} x_i^\ell - x_i & \text{if } x_i - x_i^\ell < \alpha_i \\ & \text{or} \\ -\alpha_i & \text{otherwise} \end{cases} \tag{20}$$

$$S_i^u = \begin{cases} x_i^u - x_i & \text{if } x_i^u - x_i < \alpha_i \\ & \text{or} \\ \alpha_i, & \text{otherwise} \end{cases} \tag{21}$$

where α_i is a specified maximum limit on the change in variables.

4. If S^r is sufficiently small i.e. if

$$|S^r| < \varepsilon_3 \tag{22}$$

where ε_3 is an arbitrary small variable convergence parameter then the design is considered optimal and the procedure is terminated. Otherwise define a comparison base

$$x_C^r = x_T^r + S^r \tag{23}$$

and evaluate all $C_q(x_C^r)$.

5. If $x_T^r = x_B^r$ and any

$$C_q(x_T^r) > C_q(x_B^r) \tag{24}$$

call $x_T^r = x_B^r$ otherwise call $x_B^r = x_T^r$. If any

$$C_q(x_C^r) > C_q(x_B^r) \tag{25}$$

then repeat steps 2 and 3 with α_i^r halved

6. Otherwise call

$$x_B^{r+1} = x_C^r \, . \tag{26}$$

Now if for all objective functions

$$\left.\begin{array}{l} |[C_q(x_B^{r+1}) - C_q(x_B^r]/C_q(x_B^{r+1})| \quad \text{or} \\ \\ |C_q(x_B^r + 1)| \end{array}\right\} < \varepsilon_4 \tag{27}$$

where ε_4 is an arbitrary small objective function convergence parameter the design is considered optimal and the procedure is terminated.

7. Otherwise define a new temporary base

$$x_T^{r+1} = x_C^r + \Delta(x_C^r - x_B^r) \tag{28}$$

where $\Delta \geq 0$ is a move acceleration parameter. Now increase r by one and repeat steps 2-5. Continue the process until Eq. (22) or (27) is satisfied.

This solution of DFP of Eqs. (15-21) will designated a move S which, based on local linearization appears, to be the best possible for a given specified maximum permissible change α in variable values. This DFP differs from that of Zoutendijk [17] in that it admits multiple objective functions [4] and tends to drive the design as necessary to a location estimated to be on the constraint boundary rather than tending to move the design parallel to or deflected away from this boundary [18]. The relaxation here of the usual non-negativity requirement on σ allows a design move from the infeasible region to produce an increase in objective function value in such a manner as to keep such increase at a minimum while producing movement to a location on the estimated critical constraint boundary. This allows movement in the feasible region.

The penalty function of Eqs. (11-13) is needed to allow the comparison in Eq. (24) and (25) of the desirability of a point in design space for the purposes of determining a suitable value of step size α^r needed to avoid excessively large moves and to prevent oscillation. For example for a problem as illustrated in Fig. 1 the initial α^0 would produce convergence without any need for step size reduction. For a problem such as that illustrated in Fig. 2 however oscillation about the optimum would result unless α^0 is reduced. The penalty form is used in preference to the objective function alone since a move which produces substantial constraint violation reduction with some increase in objective function value is generally more desirable than a move that produces the reverse situation. The penalty form is thus preferable for comparison.

For most engineering design problems where optimal designs are on a constraint boundary, $\Delta = 0$ is preferred since the design move will then be

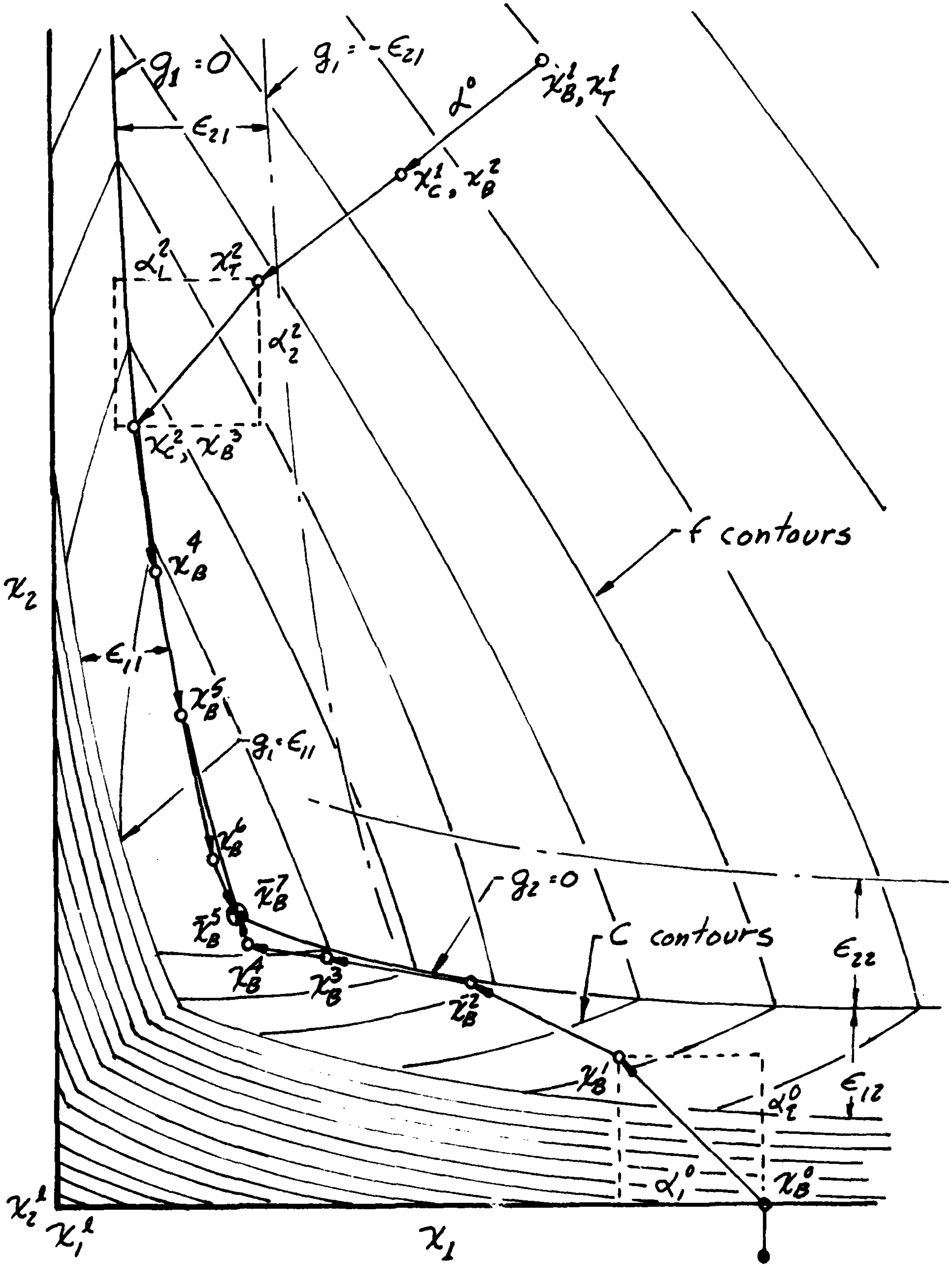


Figure 1 Synthesis Path Where the Optimum is at a Corner

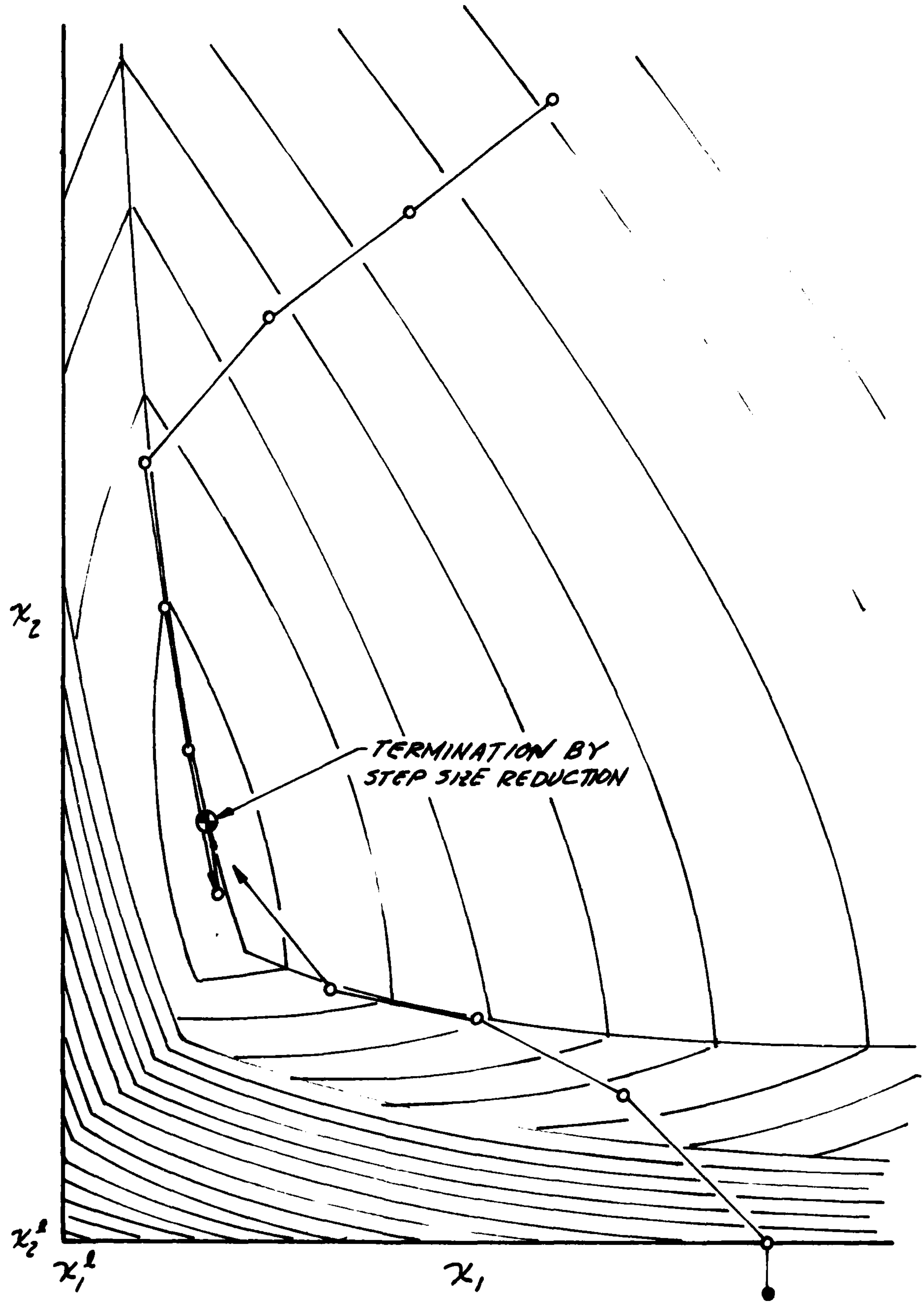


Figure 2 Synthesis Path Where the Optimum is not at a Corner

equal to the estimated best move S. However in some situations where the optimal design is unconstrained and the objective function surface is in the form of a ridge $\Delta = 0$ will not produce efficient movement since in this case the procedure reduces to an ordinary gradient search and such a search is not well adapted to such surfaces [20]. For such problems $\Delta = 1$ is recommended. Such a value of Δ produces a "pattern" [20] type ridge climbing search (modified per Section IV) a illustrated in Figure 3.

From Fig. 3 it can be seen that the move S provides the turning component for movement along a curved ridge. The basic procedure can be modified to improve such movement by utilizing the gradient information and information generated by the move. Thus where $\Delta \neq 0$ is used if the move S fails to improve the design compared to the temporary base use the gradient information at point x_T and the changes in C_q resulting from the move to construct Q quadratic approximations to C_q along a line with the direction of S. Now if there is a point along this line where on the basis of these approximations a better design can be located move to that point and call it x_c. Then continue with the procedure. This feature has been incorporated into the procedure illustrated in Fig. 3.

The problem of Eqns. (15-21) may be put into ordinary Linear Programming (LP) form by the variable transformation [18,21]

$$S' = S + S^{\ell} \tag{29}$$

$$S'^{u} = S^{u} + S^{\ell} \tag{30}$$

and $\sigma_1 - \sigma_2 = \sigma$ (31)

so that now the LP variables are S_i', σ_1 and σ_2 where

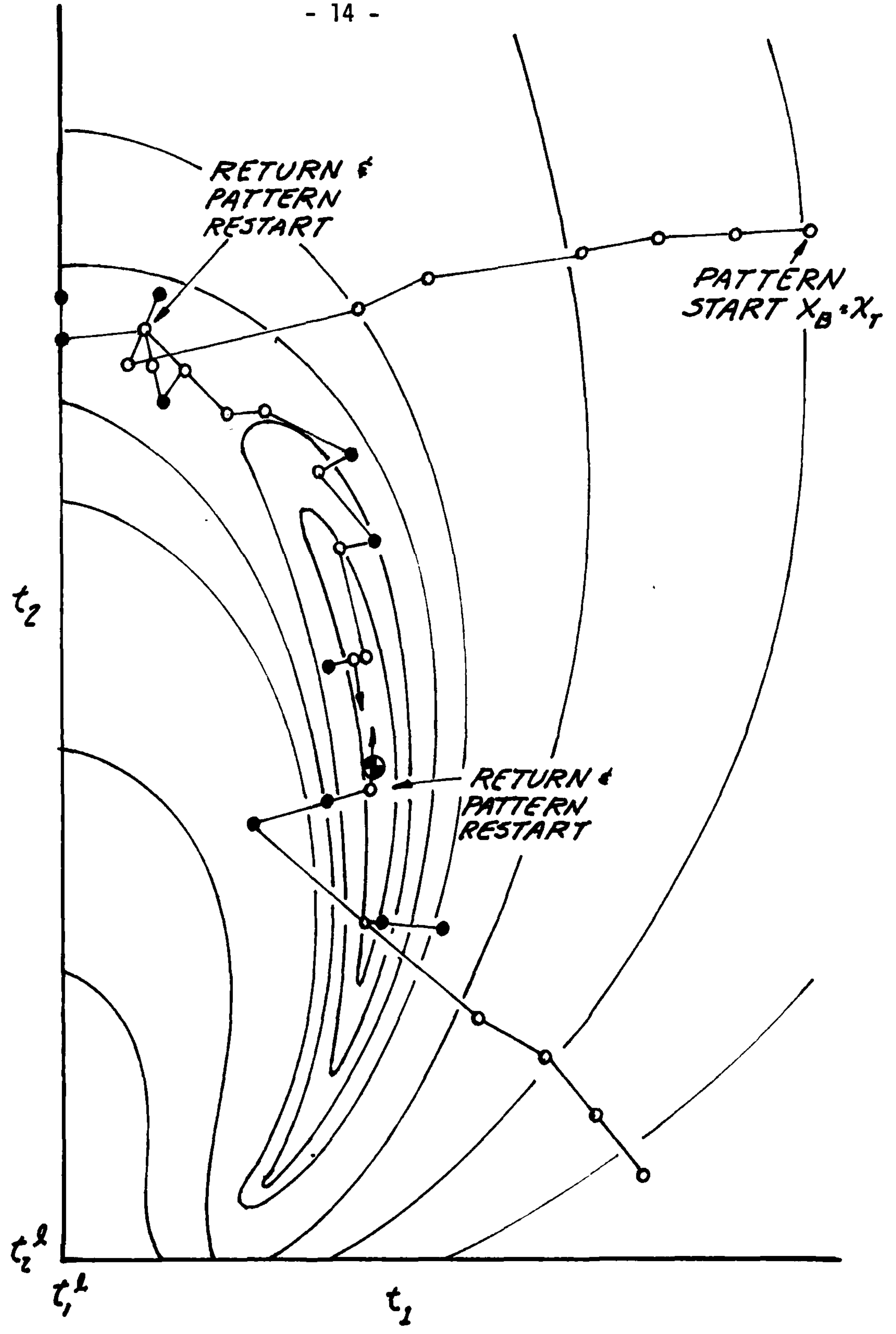


Figure 3 Synthesis Path for Unconstrained Optimum

$$\sigma_1 > 0$$

$$\sigma_2 > 0 \qquad (32)$$

$$0 \geq S_i' \leq S_i'^u \qquad (33)$$

The upper limits on S_1' can be conveniently treated by a bookkeeping procedure and need not effe t problem size or significantly increase computational effort [21].

Several factors must be weighed when evaluating a numerical optimization procedure for a particular application. Problem characteristics which should be considered are; the number and nature of the problem variables and objective and constraint functions; problem topology; computational effort associated with function evaluation and the search algorithm execution; and the admissible range of location of the initial points of the optimal search.

The procedure described above has the flexibility to efficiently treat a wide variety of problems including those with single or multiple linear and nonlinear objective and constraint functions. It includes regional constraints without significant added effort. It admits infeasible starting points without resort to a separate algorithm for such points. Furthermore it can combine the ridge climbing properties of the Pattern type searches with the rigour of the gradient based methods.

This procedure exchanges the simple derivative free evaluation of local function topology of Refs. [11 and 19] with a much more complex but also more effective gradient based method. The use of the simple local search may be desirable where the problem employs simple easy to evaluate functions since the computational effort associated with formulation and solution of the DFP may greatly exceed that associated with the need for additional function evaluations resulting from a less effective local search. The simple search can also be more desirable where the functions are not differentable. High efficiency is, however, usually not of great importance in problems employing easily evaluated functions since optimal designs can be generated in such cases at low cost with many conventional MP procedures. Optimization algorithm efficiency is of great importance in problems where design analysis is computationally demanding. Since most engineering design problems possess well behaved derivatives and since in cases where total computation is of importance this effort may be reduced by introducing optimization algorithm computation in order to minimize the number of computationally demanding objective or constraint function evaluations the added complexity of the proposed procedure is justified.

IV. MAXIMIZATION (STATE VARIABLE) PROCEDURE

In the general mini-max problem at each search point in the minimization (design) problem one must solve the state variable problem. Often the solution in known, assumed, or results directly form the determination of behavior. In such cases there is no need to search for this solution. This is true where, for example, one knows, or assumes the location of the maximum stress or if one computes buckling behavior from the matrix solution of the eigenvalue problem where the eigenvalues are automatically ordered. In other cases the solution can be obtained analytically. The case where a numerical solution to such problem is required is considered here. This section is concerned with the solution and nature of the maximization problems of Eqs. (4-6).

The choice of numerical procedure reflects the nature of the problem characteristics. For state variable optimization these are that the problem is often characterized; by a flat topology with a low ridge, frequent multiple local optima; a single objective function; only regional constraints; and relatively few variables. The feasible region is thus easily located in such problems. Furthermore in the mini-max problem in the search for the maximum the initial search point is often near the maximum point if one uses the optimum of the previous maximization as the initial search point for the new maximization problem. This latter characteristic is due to the fact that in the design variable search since behavior is often not drastically altered after a design variable move the topology of the state variable problem will likewise not be drastically changed. This is particularly true during the latter stages of the design variable search.

First consider the integer state variable problem. Ref. [12] describes a procedure for treating such problems and gives the rationale for its use in buckling constraint applications. It utilizes an integer variant of the Dichotomus search [20]. The appropriate boundaries are searched and local optima located. The interior is then searched using a sequential search with a corner check. Often this procedure requires only an evaluation of integers near the optima since the function topology is often similar to the previous search.

In the study of Ref. [16] the procedure always produced the minimum frequency. The extremely difficult buckling load surface (see Fig. 4) would, however, produce occassional failure as the design variable optimum is approached. In the neighborhood of the design variable optimum the behavior function surfaces in terms of the state variables are often characterized by the presence of more than one local behavioral maximum and a behavior surface, that although quite flat, nevertheless includes a narrow sharply curved ridge containing one of these optima [12,16]. Figure 4 illustrates a cylindrical shell buckling load surface where there are more than a dozen buckling modes within 1% of the critical value. On such surfaces there is really no reliable alternative to a regional exhaustive search. Thus, it is advisable at the point of design variable search termination to perform an exhaustive search of such functions. If there is excessive error then the general optimization procedure is restarted using an exhaustive search for the location of each such sub-problem maximum.

Structural design often involves the problem: Find the

$$\max_{\ell,n,m} \phi_{\ell}(n,m) \tag{34}$$

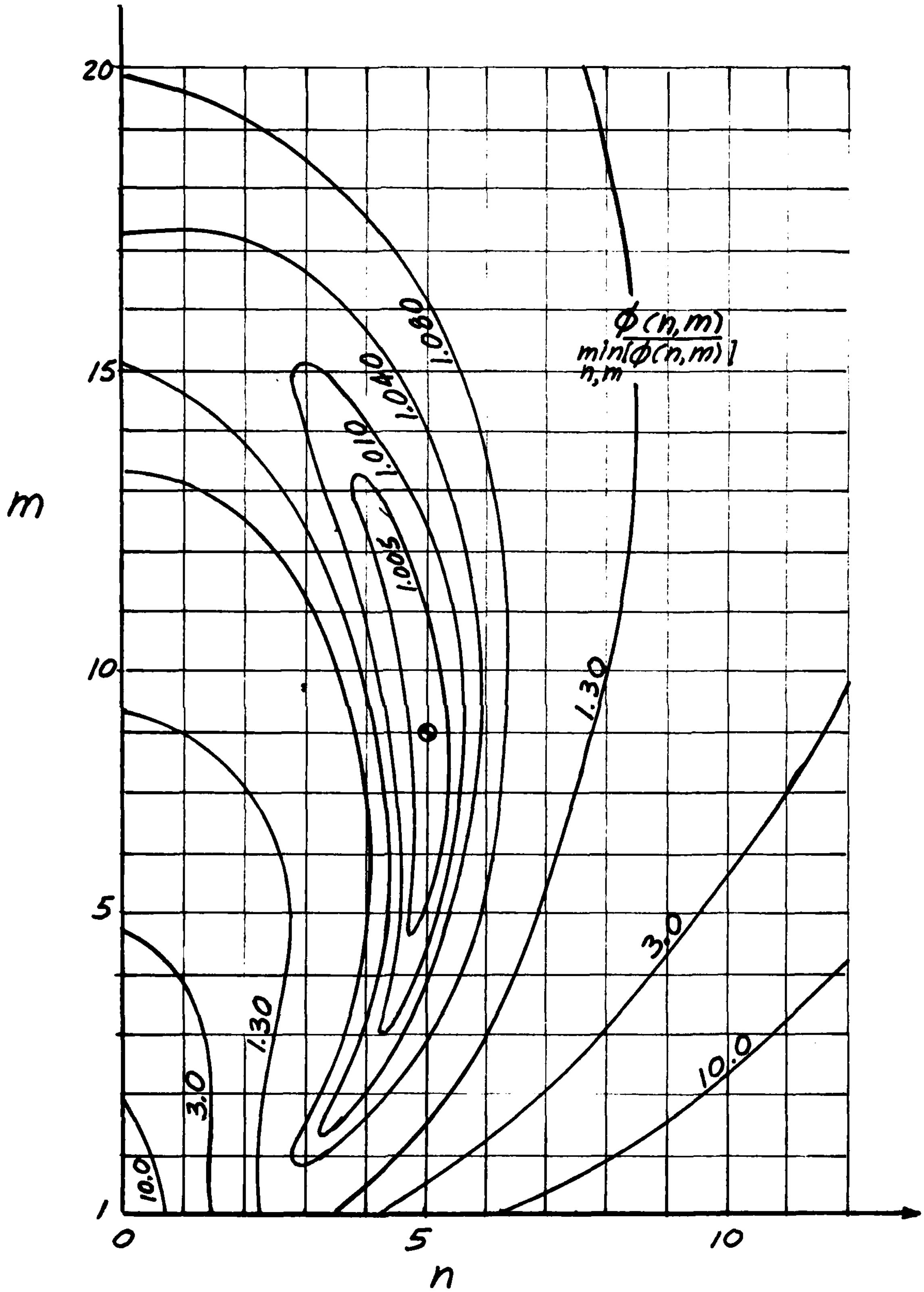


Figure 4 Typical Composite Shell Buckling Load S. face

Stiffened shell buckling or free vibration behavior are examples. Here one can identify panel, stiffener or general vibration or buckling with index $\ell = 1,2,3$ respectively where for each ℓ there exist modes associated with wave numbers n and m.

Usually an individual constraint equation is established for each mode ℓ [8,10,11,14]. However as shown in [4] and [13] one must also establish a constraint for each active or potentially active mode associated with n, and m to avoid false moves. Thus adapting the constraint designation procedure of [13] for each active integer combination ν of the state variables $t = \ell, n, m$ establish an active objective function F_a $a = 1,2,\ldots A_1$ where

$$F_\nu(x,t) - f_q(x,t) < \varepsilon_{q5} \quad q = 1,2,\ldots Q\ , \tag{35}$$

an active inequality constraint set $G_a \leq 0$ $a = A_1 + 1,\ A_1 + 2,\ldots A_2$ where

$$G_\nu(x,t) - g_j(x,t) < \varepsilon_{q5} \quad j = 1,2,\ldots J \tag{36}$$

and an active equality constraint set H_a, $a = A_2 + 1,\ A_2 + 2,\ldots A$ where

$$H_\nu(x,t) - h_k < \varepsilon_{q5} \qquad k = 1,2,\ldots K \tag{37}$$

The functions F_a, G_a and H_a $a \varepsilon A$ replace functions f_q, g_j and h_k in Eqs. (16-18) of the DFP thus allowing treatment of multiple active behavior modes. Further the zeros of the right hand side of Eqs. (16) can be replaced by the difference betwen f_q and its associated F_a allowing a larger change in F_a than F_q thereby allowing F_a to equal f_q after the move. Thus Eqs. (16) can be replaced by

$$S^T \cdot \nabla F_a^r + \sigma \leq F_a^r - f_q^r \qquad a = 1,2,\ldots A_1 \tag{38}$$

in the case of multiple active objective function behavior modes.

Now consider maximization with respect to the continuous state variables. Although the flexibility of the general minimization procedure is not needed here it is nevertheless applicable to this usually simpler problem. In fact its ability to handle surfaces with curved ridges and only regional variables quite efficiently (when using $\Delta = 1$) makes it an attractive candidate method for this problem. Furthermore, substantial simplification of a computer program for optimal design can be achieved if both the minimization and maximization techniques are similar.

Of course the complexity of the DFP is considerably reduced for the case of a single objective function with only regional constraints. For multivariable problems where regional constraints are not active ($S^{\ell} = -\alpha$, $S^{u} = \alpha$) the procedure reduces to a form of ridge climbing gradient search. Here there is no need to solve the LP problem but rather one may use $S = \alpha\nabla\phi / |\max\nabla\phi|$, where $|\max \nabla\phi|$ is the magnitude of the largest of the components of $\nabla\phi$, ϕ is the function for which a maximum is being sought and α is a step length scalar (see Fig. 3). Where there is only one variable the procedure reduces to a reasonably efficent line search without the need to formally solve the LP problem since the solution can be determined by inspection as $S = \pm \alpha$ depending on the sign of $d\phi/dt$. Furthermore by minor modification of the procedure one can eliminate the need to compute derivatives with respect to the state variables by initially assuming a sign for $d\phi/dt$ and then estimating the sign at a point by the change in ϕ produced by a given move.

V. DERIVATIVES

In the general mini-max problem a question arises as to how to treat the computation of the design and state problem derivatives and whether derivative coupling exists. For example say that one attempts to determine some stress behavior function gradients by use of finite differences. The question arises as to whether the change in control variable will produce a change in the location of the point of maximum stress and thus, after making the finite difference incremental change, whether one needs to solve again the maximization problem. Similarly, will an incremenatl change in the control variables change the value of the critical integers of the integer maximization sub-problem? A similar question arises in the evaluation of analytical derivatives.

The question of how to treat modes typical of the integer problems during differentiation with respect to the design variables is handled by associating a individual function with each active integer mode. Thus the mode use in the evaluation of derivatives is taken as the one associated with the derivative being sought. There is, therefore, no need consider the state variables in computing the design problem derivatives. The question of how to consider the continuous state variables during differentiation with respect to the design variables may be answered by observing that since the state values are optimal they are either stationary or at the region boundary during this process. Thus either the effect of change is negligible or they can not be changed. Thus the change in state variables may be ignored when evaluating design problem derivatives. The design variables are of course parameters in the state variable problem and thus changes in the control variables are not considered in evaluating state problem derivatives.

VI. ALGORITHIM CONTROL PARAMETER SELECTION

The performance of the above procedure depends on the selection of the parameters ε_{j1}, ε_{j2}, ε_3, α_i, K_1, and Δ. The test associated with ε_{j1} is needed because the equation for computing λ from the first of Eqs. (13) used to generate a reasonably symmetric "boundary ridge [19] is based on the assumption of local linearity. Thus Eq. (13) would fail to provide an appropriate penalty too far away from the constraint boundary and therefore the required values of ε_{j1} for a given j depends on the degree of nonlinearity of constraint g_j with the needed ε_{j1} decreasing as nonlinearity increases. Extensive experience with this penalty function form indicates that a value of $\varepsilon_{j1} = 0.1$ for all g_j is satisfactory where the constraints are given in a non-dimensional form

$$g_j = (B_j - U_j)/U_j \qquad U_j \neq 0 \tag{39}$$

where B_j represents the controlled behavior and U_j the upper limit on behavior even for highly nonlinear functions.

The specification of band width parameters ε_{j2} for inclusion of inequality constraints in the DFP is optional. The only purpose of these parameters is to allow the designer to reduce DFP computational effort by excluding obviously inactive constraints. This band width ε_{j2} required to avoid violation of a constraint not considered active in the DFP after a seach move depends on constraint sensitivity and step size. Several adaptive schemes are possible. The procedure of Ref. [18] uses

$${\varepsilon_{j2}}^r/\varepsilon_{j2}^o = |\alpha^r|/|\alpha^o| \tag{40}$$

The ε_{j2}^o can be arbitrarily selected where when g_j is of the form of Eq. (39).

$\varepsilon^o_{j2} = 2\eta$ is recommended where η represents an estimated fractional change in the objective function resulting from a move $|\alpha^o|$ as described below.

Alternately one can establish a value for ε_{j2} by defining these values by

$$\varepsilon^r_{j2} = F \min_i (|\alpha^r_i/g_{j,i})|\nabla g_j|^2 \qquad j \varepsilon J_p \tag{41}$$

where $\phi,_i$ designates differentiation with respect to variable x_i. All constraints not in the potentially active set J_p are ignored with respect to the DFP. This band width will, based on a linear estimate, provide with a factor of safety of F that all constraints which could be violated by a move in the ∇g_j direction where the components are limited to α^r (the worst possible move with respect to the violation of g_j) are included in the DFP. Potentially active constraints are all those within a band width double the largest band width of the constraints in the previous DFP. Initially the potentially active band width is arbitrarily selected with a value equal to 2η where the g_j are given in the form of Eq. (39) and η is as defined below. A value for F = 1 recommended since this band width will usually avoid violation of "inactive" constraints and since infeasible designs are admissible.

As a further alternative one can arbitrarily select the ε^r_{j2}, say by use of Eq. (40), and set up and solve the DFP. Potentially active constraints ($j \varepsilon J_p$) not included can be checked by determining if it appears that on the basic the gradients of these constraints they will be violated after a move [18]. Those constraints which can apparently be violated are then added to the active set and the DFP reformulated. The new DFP can be solved with relatively little effort using the basis of the former solution [21]. This process is repeated until no new constraints enter the DFP. The potentially active set can be selected as in Ref. [18].

The parameters ε_{q5} can be defined in the same fashion as the ε_{j2}. The values for α_i^o can be arbitrarily selected or using the concept of Ref. [18] in modified form by given by: All

$$\alpha_i^o = \eta f^*_q(x_B^o)|\max(f^*_{q,i})/|\nabla f^*_q|^2 \qquad (42)$$

where f_q^* is associated with q producing $\max\limits_q |\nabla f_q|$ and η is arbitrarily selected. The quantity η^r is halved so as to produce the required change in α^r needed in step 4 of the procedure of Section III. Here η is the estimated fractional change in ∇f_q^* if a move were made the ∇f_q^* direction with components limited to α^r. Thus η may be thought of as an attempted objective function reduction where $\eta = 0.5$ would be an attempt at a 50% reduction. The actual reduction would usually be substantially less than estimated since the actual move would be deflected away from the objective gradient direction by the active constraints. This would be particularly true at the latter stages of the search where the design is usually more highly constrained. Thus this formulation would have the effect of scaling down objective function changes as the optimum is approached and movement is more difficult. A value of 0.25 for η^o is recommended.

It is advisable to reduce step size in the event that a direction reversal in moving from x_T to x_c after a move from x_B to x_T produces too small a difference between x_B and x_c thus producing oscillation or a decelerated pattern movement [15]. Thus if

$$|x_B^r - x_c^r| < K_2\ |S| \qquad (43)$$

halve α^r and compute a new S until Eq. (43) is not satisfied. A value of $K_2 = 0.5$ has been found to be satisfactory after extensive experience with this and earlier procedures.

The selection of the move expansion parameter is discussed in Section III. A value $\Delta = 0$ seems preferable for most cases except where there are no active constraints where $\Delta = 1$ seems more desirable. Using the procedure of Section III it seems better in most design problems to make large moves initially and reduce move length as needed rather than using the conventional MP strategy of expanding move length based on success of earlier moves. This assumption has yet to be tested.

VII. INTERACTIVE OPTIMAL DESIGN

With few exceptions, such as the work of Michaud and Modrey [22], numerical design optimization has been employed as an "automated" design tool which attempts to locate an optimal solution without designer intervention. In most design situations, including design optimization, such intervention is desirable for a variety of reasons most of which are discussed in [22]. Efficient synthesis algorithms capable of convergence to an optimal design are well suited to interactive optimization since most redesign cycles result in substantial changes in design detail. Thus the designer can follow and guide the design process at each redesign step providing maximal utilization of designer skill without requiring a large designer effort. Such interactive optimal design is particularly desirable on computationally demanding problems where a skilled designer can reduce the total computational cost required to generate an efficient design.

Many intervention strategies can be employed for interactive optimization. The procedure illustrated in Fig. 5 is suggested here. It should be noted that the decision steps in the automated procedure coincide with corresponding decision steps in the interactive procedure, the procedure can be terminated by the operator at any time. The automated and intervention type procedures differ only in that in the intervention type additional control is given to the designer so that designer judgement which cannot yet be programmed can be used to guide the design process. Designer control can be made optional and thus a design capability based on the procedure of Fig. 5 can be fully automated. For a specific, rather than general, design programs where designer judgements required at the decision steps can be rationalized, some or perhaps even all, of these steps can be put under

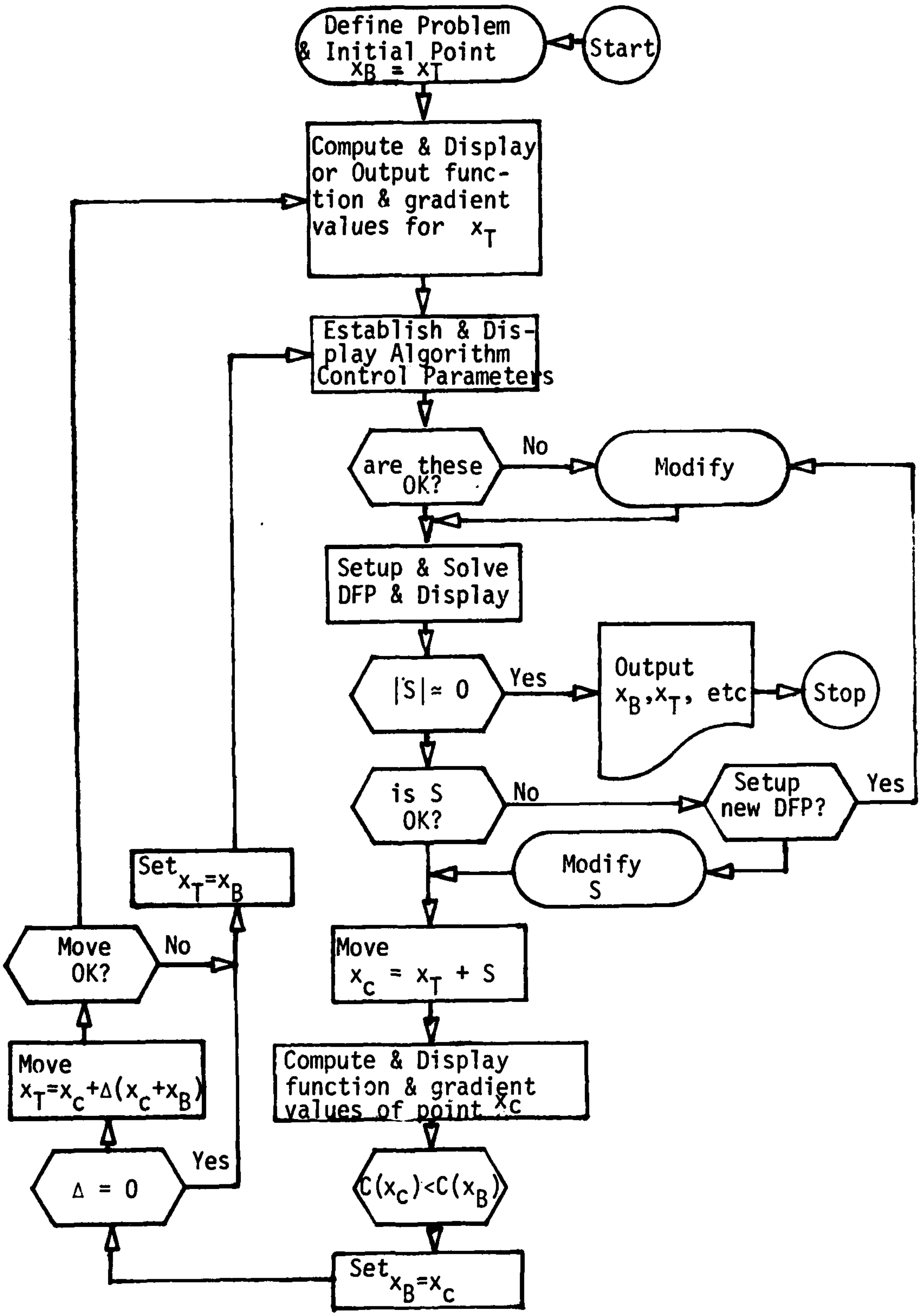


Figure 5 Designer Assisted Optimal Design

machine control thus reducing or eliminating designer intervention.

This suggested approach has yet to be tested. A substantial amount of research is needed to evolve effective means of data display and designer interrogration. Early efforts have been initiated at New Jersey Institute of Technology toward the development of an interactive optimal design capability based on the procedures described herein.

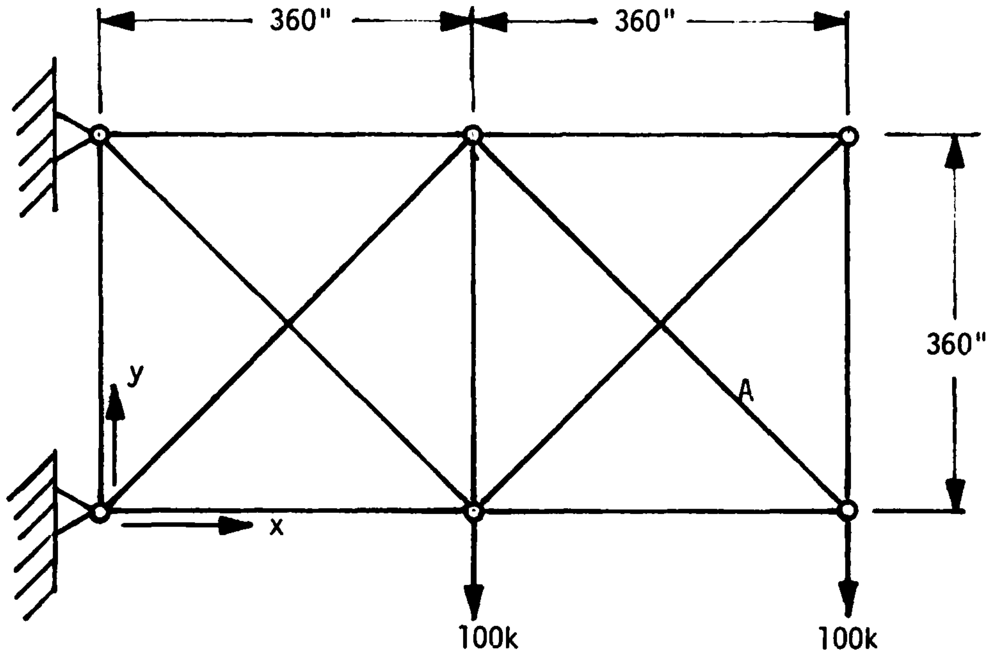
VIII. EXAMPLE PROBLEMS

A number of problems, selected to illustrate the application of the numerical optimization method given herein and to provide a preliminary test of this method, are described in this section. These problems are treated by modifying earlier computer programs used to solve these problems [4,15,18] to incorporate the new optimization method. The modified programs contain all features of the new procedure except that the upper limits on S_i are included in the DFP as additional linear constraint relations. This was done because the LP solution subroutine of this preliminary study did not include a bookkeeping procedure for handling upper limits. The added computational requirements produced by the inability of the programs utilized to efficiently treat upper regional constraints somewhat penalizes the new procedure in these preliminary studies where comparison is based on total computational effort. Accurate data for comparison on the basis of the number of function evaluations and number of design iterations is, however, provided by these examples.

Except as noted all runs used the suggested value of the algorithm control parameters given in Section VI with $\varepsilon_3 = \varepsilon_4 = 10^{-6}$ and ε_{j2} and ε_{q5} determined by Eqn. (41) and α by Eqn (42). All runs used the same parameters and starting points as the earlier studies of these examples. All except the ten bar truss problem use forward difference estimates of the gradient components.

1. Ten Bar Truss [9,18,23]

These problems are described in Fig. 6. They are now classic benchmark problems which represent a difficult challenge for numerical optimization



Problem 1 Tensile modulus 10^7 psi, specific weight 0.1lb/in^3

Variables - cross-sectional area of bars

Objective function - Truss weight

Constraints - Stress in all members to be less than or equal to 25 ksi except member A which has a 50 ksi limit.

- minimum cross-sectional area of 0.1 in^2 on all variables.

Problem 2 Same as 1 except

Allowable stress in member A is 25 ksi

Vertical displacement of all connection points (nodes) is limited to $\pm$ 2.0"

Figure 6 Ten Bar Truss Problem

methods. These problems are employed to illustrate the use of this method on a conventional MP problem and to provide a comparison of the general method with highly effective specialized structural optimization methods developed to treat such structures modeled by finite elements [9,18,23]. The stress constrained problem 1 has eight (of ten) stress constraints critical at the optimum. The displacement constrained problem 2 has two rather similar local optima. One local minimum has two critical displacement constraints and the other one critical stress constraint and one critical displacement constraint. Both problems have active regional constraints.

The procedure used for this study is similar to that of Ref. [18] except that the scaling boundary restoration of [9] is not used. Termination is by Eq. (22) or (27) with $\varepsilon_3 = \varepsilon_4 = 10^{-4}$. Further the penalty function of Eqs. (11-14) rather than weight is used for design comparison for the purpose of step size and constraint bandwidth reduction, and no non-negativity constraint on σ is used. Scaling is used only to define the initial design (as in [18]) where all members are of equal cross-sectional area. Finite elements analysis is used to determine behavior. Analytical objective function gradients are used and constraint gradients are determined by the virtual unit load method [9,23]. Runs were made with η^0 = 0.1,0.2,0.3,0.4 and 0.5 for both problems.

The new method converged to an optimum using an average of 18% fewer reanalysis cycles (11-14 reanalyses required) than the procedure of Ref. [18] on the stress constrained problem and an average of 35% fewer reanalyses for the displacement constrained problem (14-21 reanalyses required). Except for the initial and terminal points the interim designs are generally slightly infeasible. All designs are, however, considered acceptable from a practical viewpoint with violations of typically less than one percent. Improved convergence is the result of a larger weight reduction per step at a given step size and less frequent and delayed step size reduction resulting from elimination of weight increases after scaling.

2. Maximum Minimum Frequency and Maximum Frequency Separation [4]

These problems are descirbed in Fig. 7. They were selected since they illustrate the application of the new optimization method proposed herein to problems of multiple objective function form. Further they provide data for comparison of the new method with the optimization method of Ref. [4,15]. The optimal designs are characterized by many active free vibration frequency modes of similar frequency or mode pairs with similar frequency separation. In addition the maximum weight and web buckling constraints are active at the optimal designs and in the case of the minimum frequency maximization problem the shell yielding constraint is also active.

The procedure for this study is similar to that of Ref. [4] except that no pattern search is used here and the DFP of [4] does not include the improvements of Ref. [18] (improved weighting term) and those described here. In addition the method of Ref. [4] employs several ad-hoc procedures not needed with the new method. Further because of the difficulty of the problem the study in Ref. [4] uses a smaller initial step size about one tenth of that used for the present study. The integer search of Ref. [12] is used here as in Ref. [4] to locate the minimum frequency or other needed modes.

The number of function evaluations typically required to achieve the level of convergence achieved in [4] using the new procedure is approximately two orders of magnitude less than required by the procedure of Ref. [4]. An average of 710 objective and constraint function evaluations were required with the new method compared to 62,000 with the procedure of Ref. [4]. This

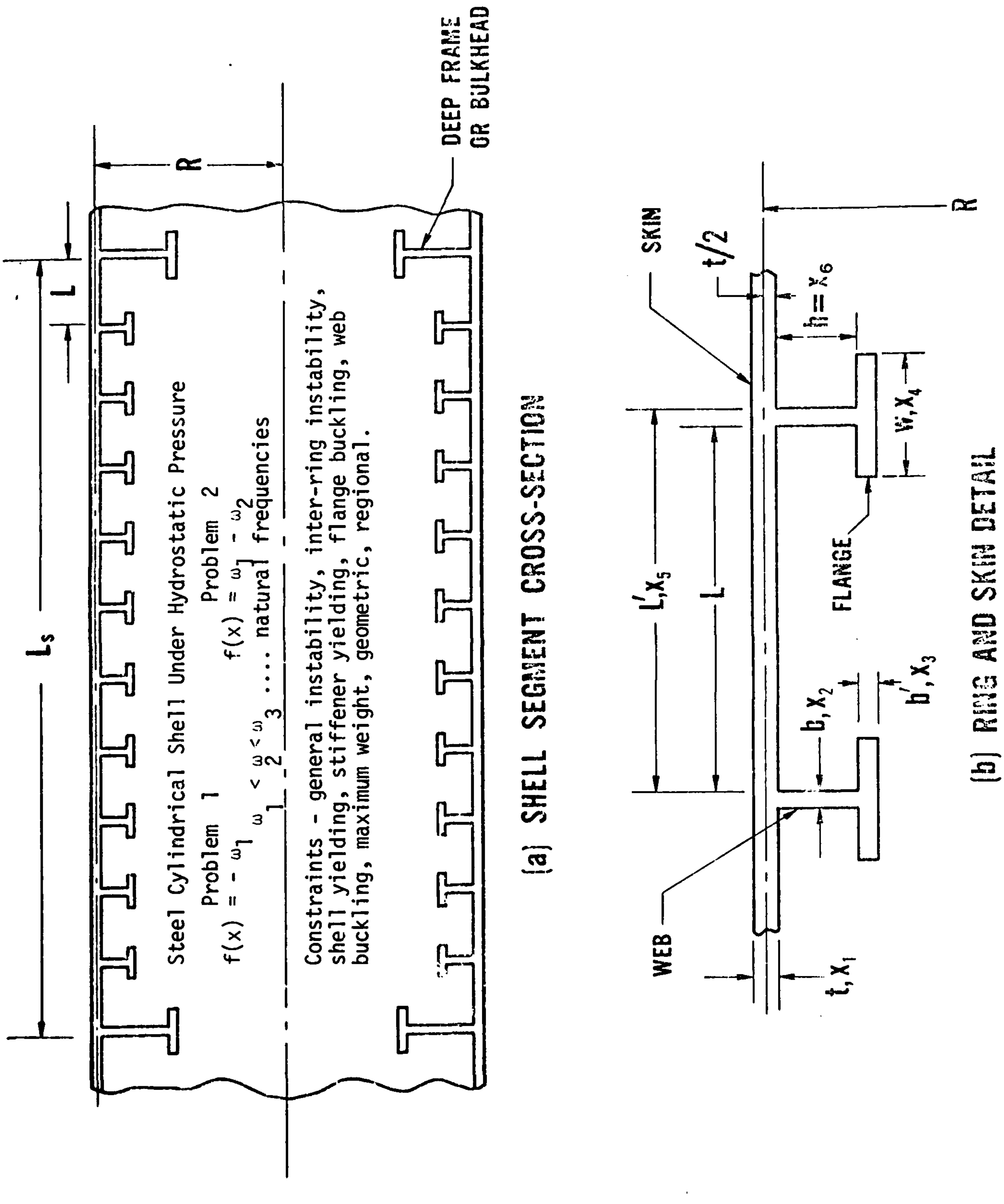


Figure 7 Shell Frequency Problems

dramatic improvement is the result of the improved DFP used here. (Note; most function evaluations are needed for derivative estimation).

3. Eason and Fenton's Test Problems [24]

The ten benchmark problems of Eason and Fenton [24] were solved using the new procedure by a computer program CADOP5 [25] developed by modifying the CADOP3 program of [15]. The problems are all relatively small (2-5 variables). Four are unconstrained at the optimum. These ten problems as a group represent a relatively difficult test set. None of the seventeen optimization codes tested in Ref. [24] solved all the problems. A typical code solved only half. Only the CADOP codes have been reported as solving all ten problems. CADOP3 [15] is similar to the procedure of Ref. [4] and differs from the CADOP5 as described above. CADOP4 [26] is similar to CADOP3 except that a "Boundary Tracking" pattern search, not using a penalty function or local exploration but rather, using a boundary restoration after a pattern move is employed in place of the pattern search of [15]. Unfortunately this boundary restoration procedure was found to be unreliable in later tests on CADOP4 failing on the 10 Bar Truss Problem.

A detailed description and analysis of the performance of CADOP5 on these problems is given in [25]. Thus only a summary is given here. When only problems constrained at the optimum are considered CADOP5 is appreciably faster than any of the codes tested by Eason and Fenton. Compared to the other CADOP codes CADOP5 is slightly faster than CADOP4 and about five times faster than CADOP3 on these problems. Using the number of function evaluations required for convergence for comparison CADOP5 typically achieved a given level of convergence on these problems with about half the number of function evaluations required by CADOP4 and roughly one tenth those required by CADOP3.

Thus even for these relatively small problems the increased computational effort associated with setting up and solving the DFP at every point was more than offset by improved search efficiency. On the four unconstrained problems CADOP5 was found to be comparable in speed to the fastest of the codes tested by Eason and Fenton. It was therefore substantially faster than the other CADOP codes on these problems. Reduction in the total computational effort associated with CADOP5 can be achieved on problems with active regional constraints by eliminating use of upper regional constraint limit equations in the DFP and employing the bookkeeping procedure of [21] for their treatment.

4. Composite Plates and Shells [16,27]

The optimization procedure described herein was also used to treat the optimal laminated filimentary composite shell study of Ref. [16] and the composite plate study of Ref. [27]. The composite shell problem is similar to the minimum freuqency maximization except that in addition to regional constraints, only a single linear constraint $\sum_{i=1}^{N} T_i \leq H$ is used, where the T_i are the thickness variable, N the number of such variables and H half the shell thickness. The composite plate problem requires the minimization of the maxima of several stress response modes. The single continuous state variable for this problem is associated with the space variable in the direction normal to the plane of the plate. The constraints are similar to the shell problem. The design variables for these problems are the layer ply angles and thicknesses.

The results of these studies is reported in Refs. [16 and 27]. In summary optimal designs in these maximum performance problems are characterized by several or even many active objective function behavior modes and thus several or many multiple objective functions. The optima are typically unconstrained. The design variable optimization procedure for this problem therefore reduces to an unconstrained multiple objective function optimal search employing a multiple gradient and pattern move strategy. Convergence typically required about 50 reanalysis cycles.

IX. CONCLUSION

The general design variable optimization procedure is found to be effective on a number of example problems. These include mini-max problems, problems with multiple objective functions as well as conventional MP benchmark problems. The potential power of the improved DFP formulation has been demonstrated in a recent study where, what is basically a less efficient version of this procedure, was found superior to the Optimality Criteria methods against which it was compared. These latter methods have received wide recognition as powerful tools for synthesis of structures modeled by finite elements [6].

This paper presents a unified treatment of the general numerical design optimization problem which appears on the basis of formulation and experience to be flexible, efficient, and robust (reliable) for a broad range of such problems. Additional studies of its performance are of course needed and additional refinement may be desirable. The procedure described herein, however, forms a relatively rigorous base for a powerful yet general synthesis capability with less resort to ad-hoc procedures often found in numerical optimization techniques. Experience with the procedure has indicated that results are not strongly dependent on initial choice of starting point or algorithm control parameters and that estimating reasonable values for these is straightforward.

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